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The Effects of Design Parameters on the Displacement and Compression Ratio of the Wankel Rotary Compressor

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ABSTRACT

The displacement and compression ratio of the Wankel rotary compressor are functions of the trochoid ratio μ and the generating ratio λ . This paper presents the influence of these two design parameters on the displacement and the maximum theoretical compression ratio. The paper also presents the governing equations for the displacement with special attention to the influence of the offset. Also presented is the relationship between the flow rate and the angular position of the rotor.

NOMENCLATURE

The following symbols are used in the development of the mathematical relationships for the gerotor:

r_1	= radius of the smaller pitch circle = $r_2(T-1)/T$
r_2	= radius of the larger pitch circle
r_C	= distance from the center of generating pitch circle to center of the generating pin
μ	= trochoid constant = $\frac{r_C}{r_2}$
r	= radius of the generating pin or offset
λ	= generating ratio = $\frac{r}{r_C}$
e	= length of crank = $r_2 - r_1 = \frac{r_2}{T}$
H	= the point of contact between the generating pin and the generated shape
N	= gear ratio = $\frac{r_1}{r_2} = \frac{T-1}{T}$
P	= instant center between rolling pitch circles, also the location of the pole
P'	= point of intersection of the line PC with the fixed pitch circle
r_E	= containment radius
S	= conjugate point on the rotor

W	= thickness of the rotor
ΔA	= area displaced by one pocket by one cycle= maximum area -minimum area
ΔV	= volume displaced by one pocket by one cycle= maximum volume -minimum volume
CR	= compression ratio= maximum volume/minimum volume
T	= number of lobes on the larger pitch circle or generating teeth
$T - 1$	= number of lobes on the smaller pitch circle
$X_f - Y_f$	= the fixed reference frame
$X_m - Y_m$	= the moving reference frame
ϕ	= input angle of the crank
ψ	= rotation angle of the moving reference frame= $\frac{\phi}{T}$
ϕ'	= input angle when the pole is coincident with P'
θ	= angle between the line PC and the X_f axis.

INTRODUCTION

The Wankel rotary compressor is a subset of the gerotor class of mechanisms. Although numerous articles have been published concerning the Wankel rotary engine, [6,8], there is little literature on the Wankel when it is used as compressor. In particular the influence of the trochoid constant and offset on the output parameters has received little attention in the literature. Presented in this paper are the influences of the trochoid constant and the offset on the displacement and compression ratio of the Wankel rotary compressor.

SHAPE GENERATION

Since the shape equations have been presented by [1,2] they are simply stated here as:

$$H_X = -e \cos \phi + r_C \cos \phi/T + r \cos \theta \quad (1)$$

and

$$H_Y = -e \sin \phi + r_C \sin \phi/T + r \sin \theta \quad (2)$$

and

$$\theta = \tan^{-1} \left(\frac{-\sin \phi + \mu \sin \phi/T}{-\cos \phi + \mu \cos \phi/T} \right) \quad (3)$$

is the angle between the X_f axis and the line PC; i.e., the line connecting the pole to the center of the generating pin, see Figure 1. The center of the generating pin, C , will be located on the positive X_f - axis when the input angle ϕ is zero. For ease in development, we substitute for the trochoid constant, μ and write Equations 1 and 2 as:

$$H_X = r_2 \left(-\frac{1}{T} \cos \phi + \mu \cos \phi/T \right) + r \cos \theta \quad (4)$$

$$H_Y = r_2 \left(-\frac{1}{T} \sin \phi + \mu \sin \phi/T \right) + r \sin \theta \quad (5)$$

For the classical Wankel rotary compressor shape, the value of T is three.

THE CONJUGATE SHAPE AND VOLUME DISPLACEMENT

Since this paper is concerned with determining the maximum theoretical values of the compression ratio, we will assume a constant cross-section of the rotor. Therefore, the volume contained in one working pocket (or chamber) is simply the cross-sectional area of the pocket times the depth of the pocket. Once the generated shape has been determined, the conjugate shape between the generating pins is required in order to determine the displacement. The method we adopt is referred to as the chamber area difference method [1]. The coordinates of the conjugate point S, relative to the moving reference frame, are

$$S_X = H_X \cos \frac{\phi'}{T} + H_Y \sin \frac{\phi'}{T} + e \cos(\phi'N) \quad (6)$$

and

$$S_Y = -H_X \sin \frac{\phi'}{T} + H_Y \cos \frac{\phi'}{T} + e \sin(\phi'N) \quad (7)$$

where

$$\phi' = \pi - \phi + 2\theta \quad (8)$$

Equations 6 and 7 were determined by translation and rotation of the point of contact onto the moving reference frame, $(X_m - Y_m)$, or reference frame from the fixed reference frame $(X_f - Y_f)$, see Figure 2. A complete development can be found in reference [2]. Substituting H_X , H_Y and ϕ' into Equations 6 and 7 and simplifying gives:

$$S_X = \frac{r_2}{T} \left\{ \cos(\phi'N) - \cos\left(\phi - \frac{\phi'}{T}\right) \right\} + r_2\mu \cos\left(\frac{\phi - \phi'}{T}\right) + r \cos\left(\theta - \frac{\phi'}{T}\right) \quad (9)$$

and

$$S_Y = \frac{r_2}{T} \left\{ \sin(\phi'N) - \sin\left(\phi - \frac{\phi'}{T}\right) \right\} + r_2\mu \sin\left(\frac{\phi - \phi'}{T}\right) + r \sin\left(\theta - \frac{\phi'}{T}\right) \quad (10)$$

Once the generated shape and conjugate shape are known, the volume displaced by a working pocket, as this pocket goes through a complete cycle from maximum volume to minimum volume, can be determined using a numerical integration scheme [6]. Since gerotors have a uniform cross-sectional area, the volume contained in a pocket is simply the thickness of the rotor, W , times the cross-section area. The cross-sectional area was presented in [1]. The area displaced by one pocket by one cycle is:

$$\Delta A = \frac{4r_2^2\mu \sin(\frac{\pi}{T})}{T-1} + \frac{rr_2}{T} \int_{\frac{\pi}{T-1}}^{\frac{\pi(T+1)}{T-1}} \left\{ \sqrt{1 + \mu^2 - 2\mu \cos(\phi N)} - \sqrt{1 + \mu^2 - 2\mu \cos\left(\frac{2\pi}{T} - \phi N\right)} \right\} d\phi \quad (11)$$

Therefore for the Wankel rotary compressor with $T = 3$, we have

$$\Delta A = 2r_2^2\mu \sin\left(\frac{\pi}{3}\right) + \frac{rr_2}{3} \int_{\frac{\pi}{3}}^{2\pi} \left\{ \sqrt{1 + \mu^2 - 2\mu \cos\left(\frac{3\phi}{2}\right)} - \sqrt{1 + \mu^2 - 2\mu \cos\left(\frac{2}{3}(\pi - \phi)\right)} \right\} d\phi \quad (12)$$

and the volume displaced is simply Δ times the thickness or:

$$\Delta V = \Delta A W \quad (13)$$

The rate of change of the flow, as a function of the crank angle is:

$$\begin{aligned} \frac{\Delta A}{d\phi} = \frac{r_2}{T} \left\{ r_2\mu \left(\cos\left(\frac{2\pi}{N} - \phi N\right) - \cos(\phi N) \right) \right. \\ \left. + r \left(\sqrt{1 + \mu^2 - 2.0\mu \cos(\phi N)} - \sqrt{1 + \mu^2 - 2.0\mu \cos\left(\frac{2\pi}{T} - \phi N\right)} \right) \right\} \end{aligned} \quad (14)$$

COMPARISON TECHNIQUES

One method of comparing Wankel rotary compressors with different values of μ and different generating pin radii is to consider the volume displaced from a contained pocket, after the pocket has cycled from the maximum contained volume to the minimum contained volume. Since we consider the Wankel rotary compressor to have a uniform cross-section comparisons of

compressors with the same thickness are possible by comparing just the cross-sectional area of one pocket [5]. Hereafter, only the pocket area will be considered. To express the results in meaningful units, the pocket displacement is divided by the area of a minimum radius circle that would inscribe the compressor, see Figure 5. This is a convenient measure of the mechanism size. The containment radius is:

$$r_E = \frac{r_2}{T} + r_C + r \quad (15)$$

To obtain a more general expression we define the generating ratio as

$$\lambda = \frac{r}{r_C} \quad (16)$$

which allows us to vary r as a function of the trochoid constant, since $r_c = \mu r_2$. Therefore, Equation 15 can be written as:

$$r_E = \frac{r_2}{T} (1 + \mu T (1 + \lambda)) \quad (17)$$

and for $T = 3$, the equation can be written as

$$r_E = \frac{r_2}{3} (1 + 3\mu(1 + \lambda)) \quad (18)$$

By scaling the compressors to fit inside a unit circle only the design parameters μ and λ need be considered since we have set $T = 3$.

CONCLUSIONS

Figures 3 and 4 show the relationship between the design variables μ and λ and the displacement and maximum theoretical compression ratio. First, it can be seen from Figure 3 that the trochoid constant μ has a significant influence on the normalized displacement. For example, when $\mu = 2.0$, ($\lambda = 0.02$), the displacement is 0.199, at $\mu = 4.0$, ($\lambda = 0.02$) the displacement is 0.16. This is a reduction in displacement of 42%. The same trend is present for $\lambda = 0.5$ where the reduction in displacement is 46% over the same range of μ .

It can also be seen from Figure 3 that an increasing trochoid constant increases the maximum compression ratio. Only the maximum theoretical compression ratio is being considered since relief pockets can be added to the rotor to decrease the compression ratio without changing the displacement [6].

Secondly, it can be seen from Figure 4 that the displacement, for a fixed trochoid constant μ , decreases for an increasing value of λ . For example, at $\lambda = 0.02$, ($\mu = 2.0$) the displacement, $\Delta A = 0.199$. Now for $\lambda = 0.5$, ($\mu = 2.0$) the displacement, $\Delta A = 0.146$. This is a 26% decrease in displacement. For $\mu = 4.0$ the decrease in displacement is 30% as λ varies from 0.02 to 0.5.

It can also be seen from Figure 4 that increasing the value of λ , for a fixed value of μ , increases the compression ratio. For example, at $\mu = 2.0$ the compression ratio increases from 15.5/1 to 16.2/1 as λ increases from 0.02 to 0.5. This is an increase in compression ratio of 4.5% accompanied by a 26% decrease in displacement. Likewise for $\mu = 4.0$ the compression ratio increases from 29.5/1 to 30/1 as λ is varied over the same range. This is increase in compression ratio of 1.67% accompanied by a 30% decrease in displacement. Figure 5 shows the generated and conjugate shape of Wankel rotary compressors with various μ and λ ratios.

In conclusion, the value of the trochoid constant has the overwhelming influence on the performance of the Wankel rotary compressor, both in terms of the displacement and compression ratio. However, the radius of the generating pin (λ) does influence the radius of curvature of the generated shape and thus the contact stress [7].

REFERENCES

- (1) Beard, J.E., Pennock, G.R., and Stanisic, M.M., "The Effects of the Design Parameters on the Generated Curvature and the Displacement of Epitrochoidal Gerotor Pumps", 1990 SAE International Congress and Exposition Paper, # 891831.
- (2) Colbourne, J.R. "Gear Shape and Theoretical Flow Rate in Internal Gear Pumps", Transactions of the Candian Society of Mechanical Engineering, Vol. 3, No. 4, pp 215-223, 1975.
- (3) Beard, J. E., Hall, A. S., and Soedel, W., "Hypotrochoidal Versus Epitrochoidal Gerotor Type Pumps with Special Attention to Volume Change Ratio and Size", ASME 1987 Design Automation Conference.
- (4) Hall, A. S., "Pin Gearing", ASME Proceedings, Mechanisms Conference, 1968.
- (5) Beard, J. E., "Kinematic Analysis of Gerotor Type Pumps, Engines, and Compressors", Ph.D. disseration, Purdue University, pp. 108-111, 1985.
- (6) Andsdale, R., "The Wankel RC Engine", A.S. Bornes and Company Publishers, New York, 1969.
- (7) Colbourne, J. R., "The Geometry of Trochoidal Envelopes and their Application in Rotary Pumps", "Mechanism and Machine Theory", 1974, Vol. 9, pp. 421-435.
- (8) Leemhius, R. S., and Soedel, W., "Vector Loop Analysis of Wankel Compressors and Engines", ASME, 78-DET-1978.
- (9) Sadler, J.P, and Nelle, D.E. "Design Analysis of an Epicyclic Rotary Pump Mechansim", ASME Journal of Mechanical Design, January 1979, Vov 101, pp. 99-107.
- (10) Schell, C.P., "Define the Geometry of the Inner And Outer Tooth Countors of a Gear Type Oil Pump", General Motors Engineering Journal, July-August-September 1969, pp. 53-54.

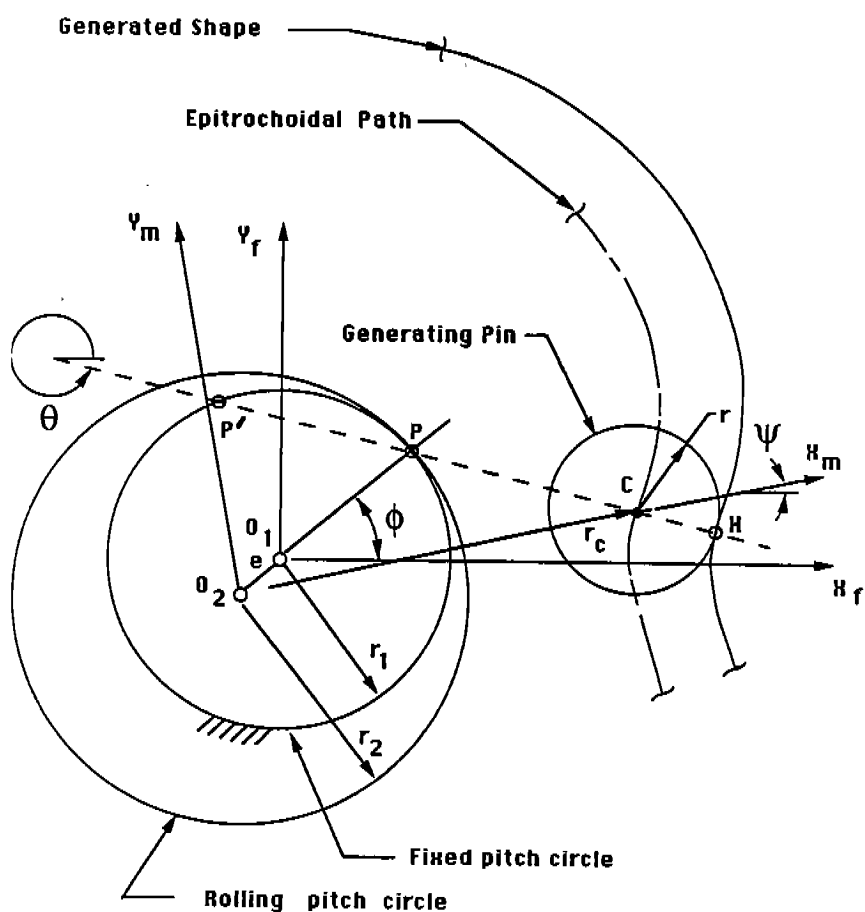


Figure 1. The generating pin and the generated shape.

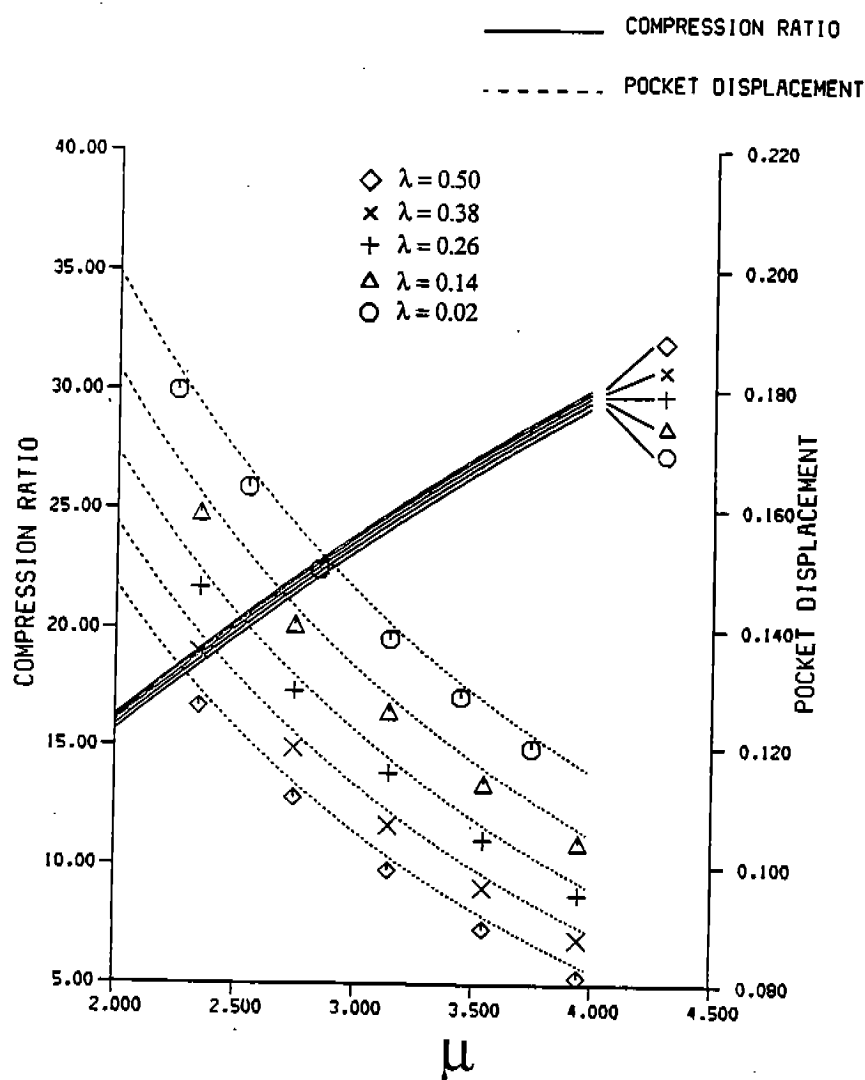


Figure 3. The compression ratio and displacement as a function of the trochoid constant, μ , for various λ values.

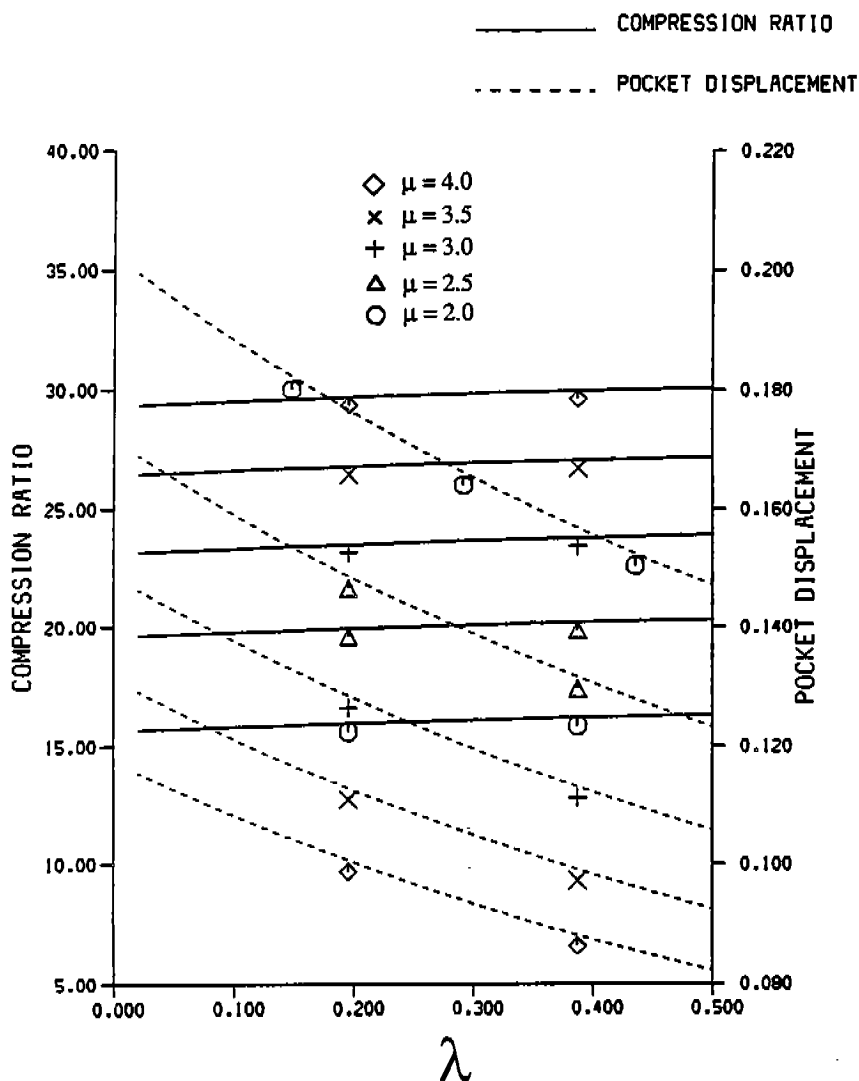
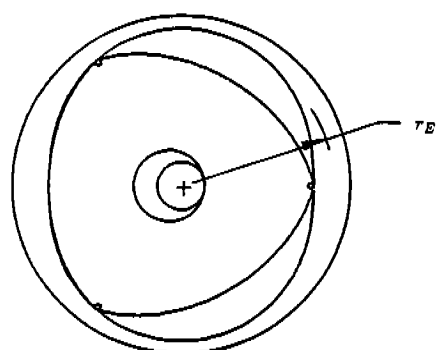
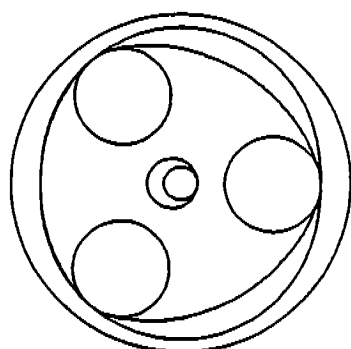


Figure 4. The compression ratio and displacement as a function of the generating ratio, λ , for various μ values.



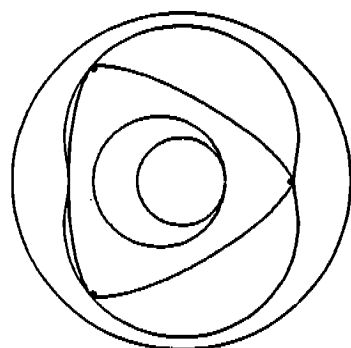
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$$\lambda = 0.02$$



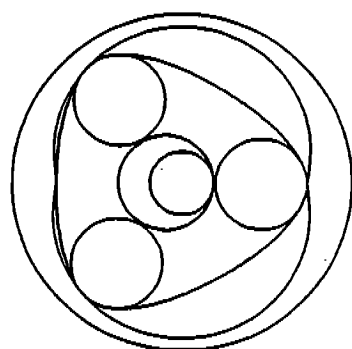
$$\mu = 4.0$$

$$\lambda = 0.50$$



$$\mu = 2.0$$

$$\lambda = 0.02$$



$$\mu = 2.0$$

$$\lambda = 0.50$$

Figure 5. Wankel rotary compressor shapes for various trochoid constants and generating ratios.